S2 Topic 1

Pythagoras' Theorem

Level: Key Stage 3

Dimension: Measures, Shape and Space
Module: Learning Geometry through a Deductive Approach
Unit: Pythagoras' Theorem

Student ability: Average

Content Objectives:

After completing the activity, students will gain experience of informally proving Pythagoras' Theorem

Language Objectives:

After completing the activity, students should be able to

- understand English terms related to the topic (e.g., *square, square root, Pythagoras' Theorem, right-angled triangle, hypotenuse, proof (n), prove (v),* and *the converse of*);
- use the key terms to state Pythagoras' Theorem (e.g., *In a right-angled triangle, the sum of the squares of the length of the two legs is equal to the square of the length of the hypotenuse*);
- use the key terms to state the Converse of Pythagoras' Theorem (e.g., *If the lengths of the three sides of a triangle ABC satisfy* $a^2 + b^2 = c^2$*, this triangle is a right-angled triangle.*); and
- follow English instructions on solving problems concerning this topic and work on related problems written in English.

Material required:

Graph paper, paper, scissors and rulers

Prerequisite knowledge:

Students should have studied the concept of Pythagoras' Theorem in Chinese and practised the related calculations.

Time: 2 lessons (2 x 40 minutes)

Procedure:

Lesson 1:

1. The teacher should teach students the meaning of the relevant mathematical terms (shown at the top of the worksheet), demonstrating the pronunciation of the terms in English.

2. Assuming that students have learnt the concept of Pythagoras' Theorem in Chinese, students should then be guided to explain the theorem in English.

3. The teacher should ask students to form groups of 4-5 and complete proof 1. Then, the teacher can ask the students to present their findings.

Lesson 2:

1. The teacher should ask students to cut 4 identical right-angled triangles (using the graph paper) and use those figures to explain Pythagoras' Theorem.

2. Students should be ready to explain their proof to other classmates using the terms they learned in the previous lesson.

3. Students should be given time to prepare the presentation with their group-mates beforehand.

Explanatory Notes for Teachers:

1. The aim of this teaching material is to give students the opportunity to have hands-on experience of mathematical proof and to practise their presentation skills related to the topic of Pythagoras' Theorem. It is therefore expected that the lesson will be conducted after the study of the topic through the medium of Chinese.
2. The teacher should ask students to work in a group (4-5 students) for the hand-on experience of cutting the papers, discussing the possible proofs and preparing for the presentation (Activity 2, proof 1). In this case, students collaborate to assign tasks within their groups and help each other to work out the proof.

3. In proof 1, the sizes of the squares can be different for different groups. Thus, the teacher can explain to students that this theorem can work for any size of triangle. For the same reason, the identical triangles can also be of any size.
4. Students can practise delivering the presentation before they are actually called on to do so in front of the class. They can also prepare, for example, by writing notes first. This will help them overcome any fear they may have of speaking English in front of a group.
5. The teacher should provide support to groups if they have any difficulties.
6. It is important that the teacher should be flexible in adjusting the teaching schedule to suit the needs of students. The teacher can arrange 2 single lessons or a double lesson for this. Students could finish all the parts (i.e. thinking, discussion and presentation) during the lesson time. On the other hand, the teacher can ask students to try out proof 2 at home as homework and then present it during the second lesson. However, for a double lesson, students may not have time to try out proof 2 and write down the presentation beforehand. In this case, the teacher has to provide more time for group discussion. Moreover, depending on the ability of the students, the teacher can ask them to hand in individual assignments of proof 2 (and skip the second presentation).

Secondary 2 Extended Learning Activities for Mathematics
Pythagoras Theorem

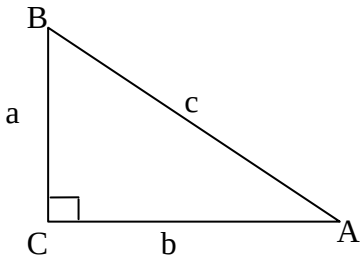
Name: _____ Class: _____( )

ACTIVITY 1: Revisiting Pythagoras' Theorem

Vocabulary:

Square 平方	Square root 平方根	Pythagoras' Theorem 畢氏定理
Hypotenuse 斜邊	Proof(n) / prove (v) 証明	Converse 逆

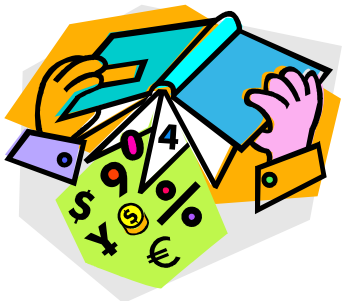
Write down Pythagoras' Theorem (using symbols):



Complete the following description of Pythagoras' Theorem:

In a _____ triangle, the sum of the _____ of the two legs is equal to the _____ of the _____.

Write down the converse of Pythagoras' Theorem (using symbols):



Complete the following description of the converse of Pythagoras' Theorem:

If the _____ of the three sides of a triangle ABC satisfy _____, this triangle is a _____.

ACTIVITY 2: Proof

Proof 1

1. On the graph paper provided, draw two squares of different sizes next to each other (Fig. 1).
2. As shown in Fig. 2, find the point on the base of the large square the distance of which from the left of the large square is equal to the length of the small square.
3. Join the lines as shown in Fig.2.
4. Cut the figure into three parts, I, II and III. (Fig. 3)
5. Try to form a square using the three parts.
6. Stick the square formed on the paper provided.
7. Label the sides of the squares and triangles.

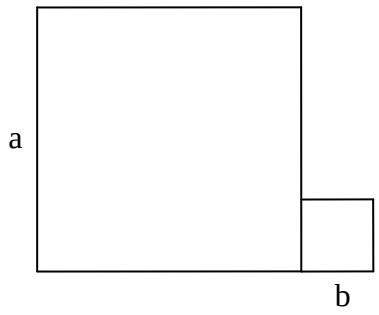


Fig. 1

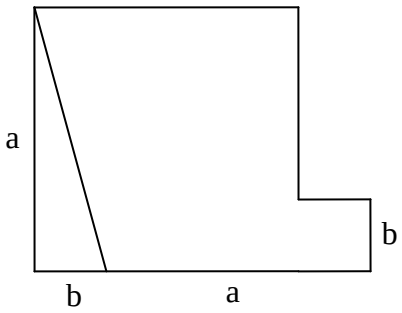


Fig. 2

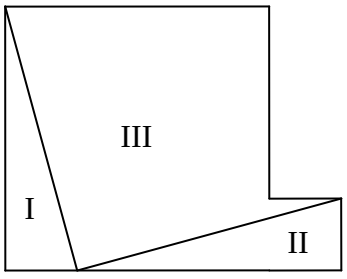
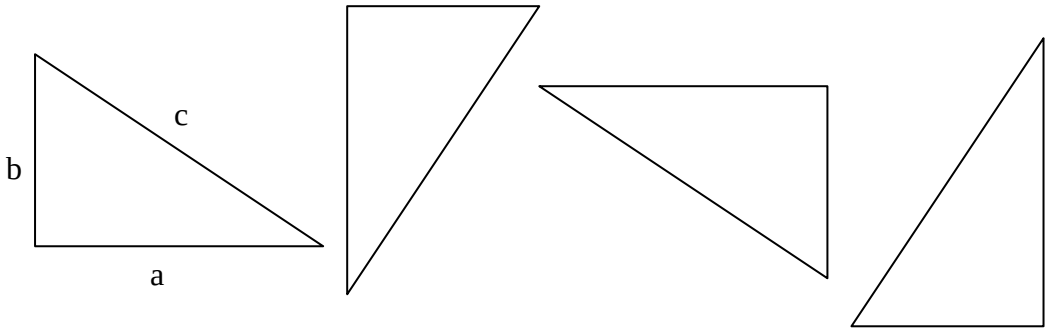


Fig. 3

8. Explain how you can use the areas of the squares to prove Pythagoras' Theorem:

Proof 2

1. Draw 4 identical right-angled triangles on the graph paper (they can be of any size).
2. Cut them out and then label the sides a, b and c as shown.



3. Try to form one square using these 4 triangles.
4. Stick the square formed onto the paper provided.
5. Explain how you can use the squares to prove Pythagoras' Theorem:

Suggested answers for teacher:

Activity 1

Write down Pythagoras' Theorem:

or

In $\triangle$ABC, $\angle$C =90°

$a^2 + b^2 = c^2$

or

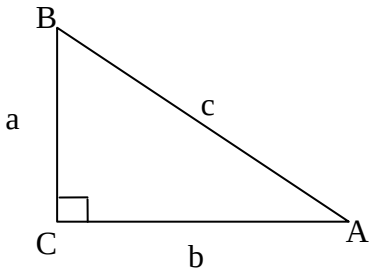
In $\triangle$ABC, $\angle$C =90°

$AB^2 = AC^2 + BC^2$

Complete the following description of Pythagoras' Theorem:

In a right-angled triangle, the sum of the squares of the two legs is equal to the square of the hypotenuse.

Write down the converse of Pythagoras' Theorem:



If $a^2 + b^2 = c^2$, then $\angle$C =90°

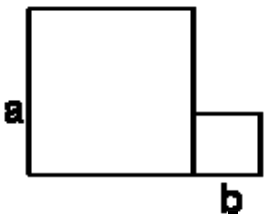
Complete the following description of the Converse of Pythagoras' Theorem:

If the lengths of the three sides of a triangle ABC satisfy $a^2 + b^2 = c^2$, the triangle is right-angled at C.

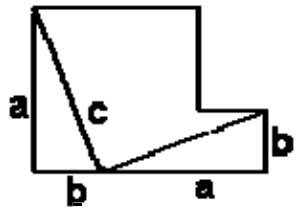
Activity 2

Proof 1

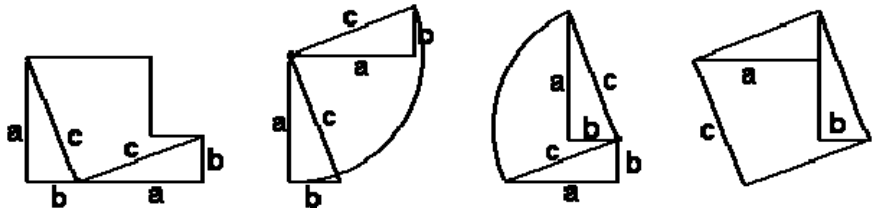
Step 1: Construct two squares with sides **a** and **b**, respectively, placed side by side. The total area of the two squares is $\mathbf{a^2+b^2}$.



Step 2: Draw 2 lines with the same length as follows:



Step 3: cut the 2 triangles out and then rotate them to form a square.

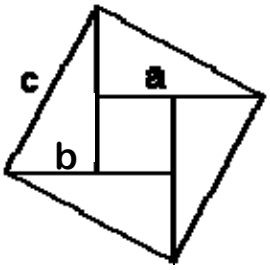


- Teacher shows that: the area of the resulting square would be $\mathbf{c^2}$

Thus, it shows that $a^2+b^2=c^2$

Proof 2:

Possible outcome 1:

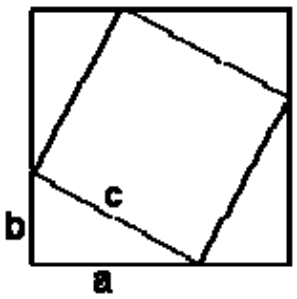


$c^2 = (a - b)^2 + 2ab$

$= a^2 - 2ab + b^2 + 2ab$

$= a^2 + b^2$

Possible outcome 2:



$(a + b)^2 = 4\times ab\div 2 + c^2$

$a^2 + 2ab + b^2 = 2ab + c^2$

$a^2 + b^2 = c^2$

Reference:

www.cut-the-knot.org/**pythagoras**/index.shtml

Graph paper